Effects of Leading-Edge Vortex Flow on the Roll Damping of Slender Wings

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The leading-edge suction analogy of Polhamus, which has proven useful for lift and drag predictions, has been extended in an effort to understand the vortex-flow effects on the roll damping of slender wings at subsonic speeds. Comparisons with experiment indicate that the method can be useful in the prediction of the large increase in roll damping with angle of attack associated with leading-edge vortex flow of highly swept, low-aspect-ratio wings. In addition, the newly developed forced-oscillation roll mechanism, suitable for testing at subsonic and transonic Mach numbers, has been used to obtain subsonic roll damping data on a series of 74° leading-edge sweep wing planforms. These data illustrate the effects of vortex flow on the damping-in-roll parameter of delta, arrow, and diamond planforms.

Nomenclature†

A = aspect ratio
b = wing span
C_l = rolling-moment coefficient
C_L = lift coefficient
$C_{l_{\dot{\beta}}}$ = rate of change of rolling moment due to rate of change of sideslip, $\partial C_l/\partial(\dot{\beta}b/2V)$
C_{l_p} = rate of change of rolling moment due to roll rate, $\partial C_l/\partial(pb/2V)$
$C_{l_p} + C_{l_{\dot{\beta}}} \sin\alpha$ = damping-in-roll parameter, body axes
C_n = yawing-moment coefficient
C_N = normal-force coefficient
C_{n_p} = rate of change of yawing moment due to roll rate, $\partial C_n/\partial(pb/2V)$
C_S = suction-force coefficient
C_T = thrust-force coefficient
C_{T_p} = rate of change of thrust due to roll rate, $\partial C_T/\partial(pb/2V)$
C_Y = side-force coefficient
C_{Y_p} = rate of change of side force due to roll rate, $\partial C_Y/\partial(pb/2V)$
$(K_{l_p})_V$ = increment in roll damping due to vortex flow (see Fig. 6)
M = Mach number
p = roll rate
$pb/2V$ = nondimensional roll rate
V = freestream velocity
α = angle of attack
β = angle of sideslip
Λ = wing leading-edge sweep angle
ω = oscillation frequency
$\omega b/2V$ = reduced frequency
$(\ )_P$ = coefficient determined by potential flow theory

Introduction

THE history of transport aircraft has been characterized, in general, by a continued effort to attain higher and higher speeds so as to increase their productivity and economy. This quest for increased speed has resulted in both configuration and operating trends which have necessitated a renewed effort with regard to the prediction of dynamic-stability derivatives. This is illustrated in Fig. 1. Here are shown representatives of a subsonic propeller transport, a subsonic jet transport, and a supersonic transport. The vertical scale represents the limiting angle of attack dictated by various aerodynamic low-speed considerations, and the horizontal scale is the cruise speed. The aircraft sketches illustrate the configuration changes and show the large increases in wing sweep with increasing cruise speed. The limiting angle of attack has progressed from about 12° for the propeller aircraft having high-aspect-ratio unswept wings to greater than 20° for the slender Concorde supersonic transport. At these extremes of angle of attack and angle of sweep, many of the stability derivatives have characteristics which are completely different from those normally encountered in low-speed designs. The flow over thin highly-swept wings at Mach numbers below that corresponding to a sonic leading edge is generally characterized by a leading-edge separation vortex pattern. This vortex pattern is apparent if the angle of attack differs only slightly from the design angle of attack corresponding to the wing camber. The vortex strength increases markedly as this angle-of-attack difference is increased. Potential flow theories have been found to be grossly inadequate for predicting the lift and drag characteristics of wings exhibiting leading-edge vortex flow, and it is anticipated that prediction of the dynamic-stability characteristics and handling qualities of aircraft employing such wings will be correspondingly difficult. A research program has, therefore, been initiated at NASA Langley Research Center aimed toward developing an understanding of lead-

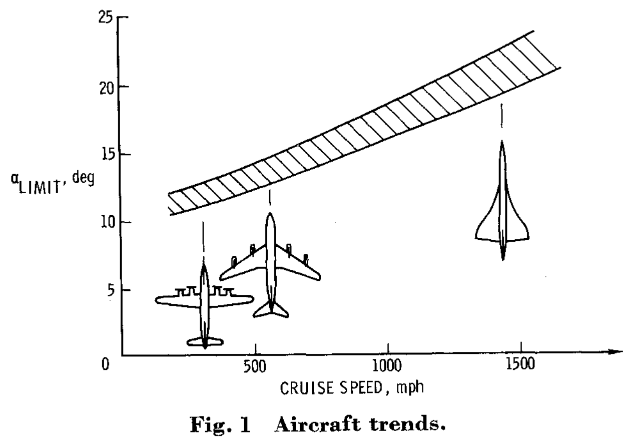

Fig. 1 Aircraft trends.

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† All aerodynamic coefficients are nondimensionalized by dividing the forces by the product of the dynamic pressure and the wing planform area and the moments by the product of the dynamic pressure, wing span, and planform area.

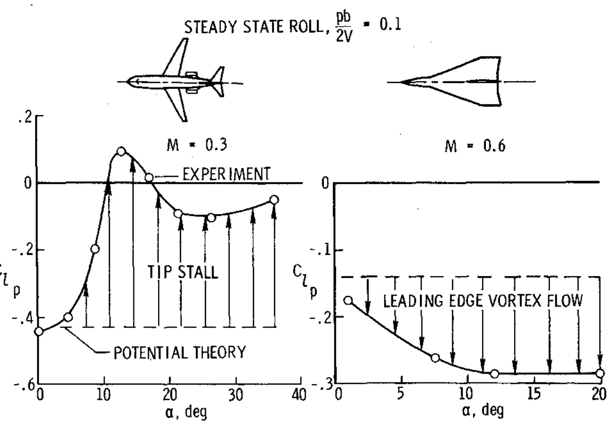

Fig. 2 Roll damping trends.

ing-edge vortex effects on dynamic-stability derivatives and development of prediction methods using theoretical concepts that have shown some success with regard to lift and drag prediction. The particular derivative to be discussed in this paper is the roll damping. This derivative is of primary importance in regard to the roll response and the maximum roll rate that can be developed by an aircraft.

Discussion

Effect of Wing Sweep on Roll Damping

The measured roll damping (C_{l_p}) from steady-state roll tests for two configurations having grossly different wing sweeps is shown in Fig. 2 and it will be noted that the angle-of-attack trends are entirely different. As shown on the left of the figure, the roll damping for a transport configuration with a leading-edge sweep of 28° decreases with increasing angle of attack until the roll damping actually changes sign.[1] This decrease in roll damping with increased angle of attack is primarily a result of tip stall or separation on the outboard sections of the wing. At angles of attack greater than about 17°, the roll damping of this configuration recovers a portion of its original positive damping (negative values of C_{l_p}).

The data on the right-hand side of Fig. 2 are for a variable-sweep fighter model.[2] In the 72.5° leading-edge-sweep configuration, the roll damping is seen to increase with angle of attack up to about 12° and then remain at that level until at least an angle of attack of 20°. This increase in roll damping with angle of attack is attributed to leading-edge vortex flow in which the flow separates from the leading edges and rolls up into two spiral vortex sheets. These vortex sheets lie above the wing leading edge and slightly inboard of the leading edge and provide increases in lift, above that for the attached flow condition, and corresponding increases in roll damping. The lift increase is usually referred to as "nonlinear" or vortex lift.

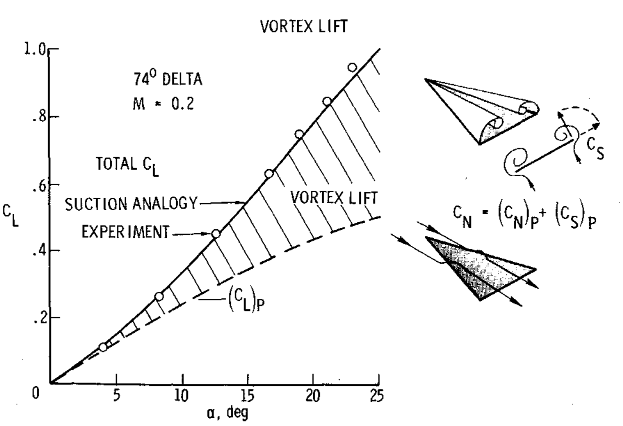

Fig. 3 Leading-edge suction analogy.

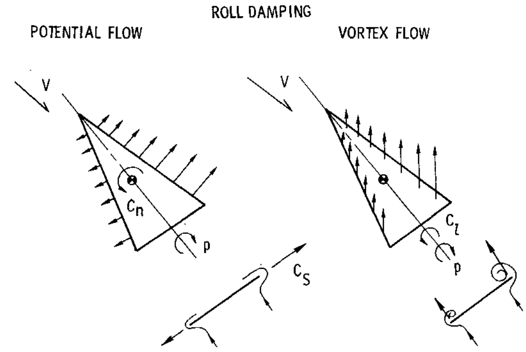

Fig. 4 Leading-edge suction analogy.

Leading-Edge Suction Analogy

A considerable amount of work has been done relative to the development of analytical methods for predicting the vortex lift of slender wings. However, because of the difficulty involved in accurately representing by a mathematical model the complicated flowfield associated with the separation induced vortices, most methods have been based on oversimplified approximations. A notable exception is the method used by Smith[3] which is based on a rather detailed mathematical model of the vortex flow. However, the method of Ref. 3 is based on slender body theory which violates the trailing-edge Kutta condition and therefore is not suitable at subsonic speeds. In addition, this method has never been applied to the rolling mode, to the author's knowledge. A method, based on intuitive reasoning, which appears to successfully circumvent the various problems has been developed by Polhamus.[4-6] The concept is based on a leading-edge suction analogy and is illustrated in Fig. 3. The basic assumption of the leading-edge suction analogy is that with the reattached flow induced by the vortex, the normal force acting on the wing upper surface required to maintain the flow over the vortex is the same as the leading-edge suction force required in potential flow. Therefore, the total normal force acting on the wing is the sum of the normal force predicted by potential flow plus the leading-edge suction force predicted by potential flow. The analogy provides an easy and accurate method of predicting the vortex lift for delta-like wings over a range of angles of attack, aspect ratio, and Mach number. A comparison of the predicted lift and the experimentally determined lift for a 74° leading-edge sweep delta with sharp leading edges is shown on the figure. The experimental data shown here were recently obtained by E. E. Davenport of the Langley Research Center. As can be seen, the agreement is very good and similar agreement over a wide range of wings has been published.[4,5]

It appeared possible that the leading-edge suction analogy might be useful in predicting the effects of leading-edge vortex flow on the roll damping of sharp-edged highly swept wings and this application of the concept is shown in Fig. 4. On the left of this figure, the potential flow leading-edge suction forces are shown acting on a rolling delta wing at an angle of attack. As a result of the combined angle of attack and rolling pressure distributions, an asymmetrical suction-force distribution is developed and a negative yawing moment is generated which produces the C_{n_p} derivative (the yawing moment due to roll rate). If we consider that this is a sharp-edged wing that physically will develop no leading-edge suction and that the resulting vortex flow will induce reattachment on the upper surface, then the leading-edge suction analogy suggests that the suction forces should be rotated

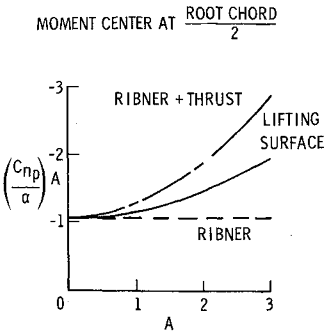

Fig. 5 Potential flow C_{n_p} for delta wings.

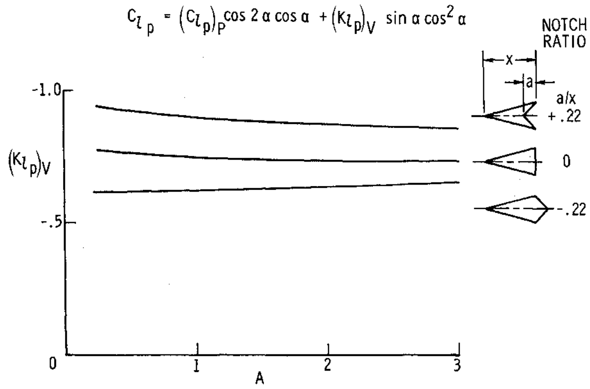

Fig. 6 Roll damping increment due to vortex flow for slender wings.

about the leading edge, which will result in an asymmetrical normal-force loading. This asymmetrical normal-force loading contributes an effective rolling moment due to roll rate or an effective roll damping from the leading-edge vortex flow. Evaluation of this damping component by use of potential flow theory should make it possible to predict the increment of roll damping resulting from leading-edge vortex flow as a function of angle of attack. In an interesting study, Harvey[7] measured the static pressures on and total pressures near the surface of an 80° sweep delta wing during steady roll at zero angle of attack which verified the existence of the vortices inboard of the leading edges and indicated that the vortices grew in a nonconical manner from the apex of the delta wing. This is a result of the local angle of attack varying with the local semispan

$$[\alpha = \tan^{-1}(py/V)] \tag{1}$$

In order to use the leading-edge suction analogy to predict the increment in roll damping due to vortex flow, an accurate method must be used to determine the distribution of the leading-edge suction force under combined conditions of angle of attack and rolling. Lifting-line theory is not adequate for prediction of the correct spanwise variation of the leading-edge thrust of swept wings, since it does not properly account for induced camber effects.[8,9] For this reason, it was necessary to use a lifting-surface method to compute the distribution of the leading-edge suction force due to rolling. The subsonic lifting-surface digital program by Wagner[10] which already included the computation of the suction-force distribution for the symmetric angle-of-attack condition was extended to the combined angle of attack and rolling condition. This allowed the roll damping and the yawing moment due to roll rate to be computed. The work of modifying and extending the Wagner program was done by J. E. Lamar of the Langley Research Center.

As a check on the accuracy of the distribution of the leading-edge suction force in the very slender-wing range, a comparison of the lifting-surface solution for the yawing moment due to roll rate C_{n_p} was made with the classic slender-wing solution of Ribner[11] and is shown in Fig. 5. Ribner's original slender-wing theory was only applicable for aspect ratios less than one-half. In a subsequent paper, Ribner[12] extended his work to larger vertex angle delta wings and included the effect of the thrust component of the suction force on the C_{n_p} derivative. This result is indicated in Fig. 5 as "Ribner + Thrust." Several items of interest can be noted in this figure. First, it can be seen that the lifting-surface solution for C_{n_p}/α properly approaches the slender-wing solution as the aspect ratio approaches zero. Second, it is apparent that the slender-wing theory, even with the thrust term included, is inadequate in its prediction of C_{n_p}/α in the aspect-ratio range of current practical interest. This inadequacy is a result of the slender-wing theory not satisfying the trailing-edge Kutta condition. Based on this, it is apparent that lifting-surface theory must be used to determine the distribution of leading-edge thrust for application of the suction analogy.

The modified Wagner lifting-surface program was used to determine the increment to the roll damping due to vortex flow through the use of the leading-edge suction force and its spanwise distance from the axis of symmetry. The values of the predicted increment in roll damping due to vortex flow for low-aspect-ratio wings by use of the leading-edge suction analogy is shown in Fig. 6. There is little variation in the vortex-flow increment in roll damping with aspect ratio. There is, however, a planform effect as shown by the notch ratio series of delta, diamond, and arrow planforms. The total roll damping of a slender wing with sharp leading edges is assumed to be the sum of the potential flow value and the increment due to vortex flow. The equation for the total roll damping in stability axes is

$$C_{l_p} = (C_{l_p})_P \cos 2\alpha \cos\alpha + (K_{l_p})_V \sin\alpha \cos^2\alpha \tag{2}$$

where $(C_{l_p})_P$ is the potential flow value of the roll damping, determined herein by the Wagner lifting-surface program, and $(K_{l_p})_V$ is the increment in roll damping due to vortex flow by use of the leading-edge suction analogy. The trigonometric terms have been retained so as to extend the applicability of the equation to high angles of attack. The total angle of attack was taken as the sum of the geometric angle of attack and the angle of attack due to the rate of roll. The angle of attack induced by the rolling motion was assumed small in comparison to the geometric angle of attack.

In Fig. 7, a comparison is made of the experimental values of roll damping as determined for a variable-sweep aircraft model with the wing in the 72.5° sweep position[2] and the predicted values using the leading-edge suction analogy to give the vortex-flow increment to the roll damping. The modified Wagner lifting-surface program was used to determine the potential flow roll damping and the vortex-flow contribution (by means of the suction analogy) for the total wing plus horizontal-tail planform. The over-all planform was simpli-

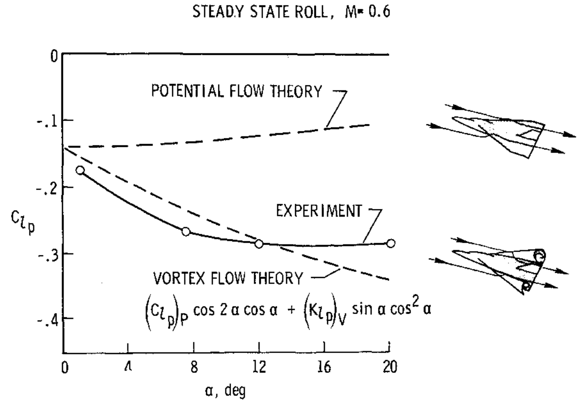

Fig. 7 Comparison of theory and experiment.

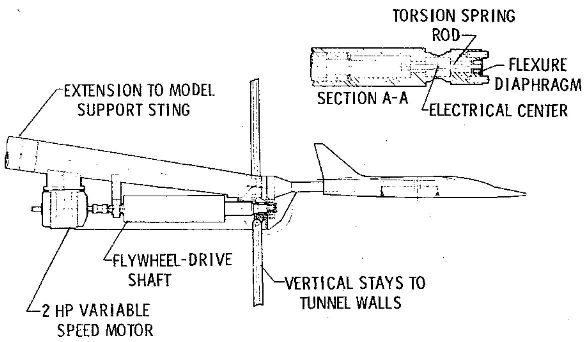

Fig. 8 Forced-oscillation roll mechanism.

fied by extending the wing leading edge to the model center line and by using streamwise tips. The vertical-tail contribution is not included in the theoretical estimate because of the unknown interference effects of the wing and fuselage on the vertical tail. The difference between the theoretical and experimental curves in the low angle-of-attack range is relatively small and may be due entirely to the vertical-tail contribution. The agreement in the trends of the experimental data and the theory using the suction analogy is considered good and illustrates the importance of including the vortex effect. If the theoretical curve were shifted to account for the probable vertical-tail contribution, it would then appear to form the upper limit in magnitude of the roll damping. The difference in the theoretical and experimental curves in the high angle-of-attack range is thought to be an inboard movement of the vortex cores with angle of attack which would decrease the experimental roll-damping increment due to vortex flow because of the reduced moment arm.

It should be mentioned that while the model wing of Ref. 2 was relatively thin, it did not have a sharp leading edge. However, analysis of the lift characteristics have indicated that leading-edge separation and the resulting vortex flow occurred on this wing at a relatively low angle of attack. The results presented in Fig. 7 indicate that the effect of the vortex flow can be very large and that the suction analogy appears to be useful in predicting the general magnitude of the effect.

Forced Oscillation Roll Mechanism

The discussion so far has dealt with the steady-state damping-in-roll term only. However, in actual aircraft motion the effects of the derivative $C_{l\dot{\beta}}$ can also be important. A new mechanism developed at the Langley Research Center for the investigation of the damping-in-roll parameter about body axes ($C_{l_p} + C_{l\dot{\beta}} \sin\alpha$) of winged configurations at subsonic and transonic speeds is sketched in Fig. 8. This is a forced-oscillation-in-roll sting and balance combination that is compatible with either the 7 × 10-ft high-speed tunnel or the 8-ft transonic pressure tunnel at the Langley Research Center. The model is rigidly forced in a fixed $2\frac{1}{2}°$ amplitude oscillation about the model body axis at a variable frequency. The principles of operation are the same as those previously used for measuring dynamic-stability derivatives in pitch or yaw with small amplitude, rigidly forced oscillation systems.[13,14] A 2-hp variable-speed motor is used to oscillate the sting and model by means of an offset crank to give essentially sinusoidal motion. A mechanical spring in the form of a torsion spring internal to the sting is connected to the front of the strain-gage balance section and is used to provide a restoring torque. This allows the model to be oscillated at the frequency for velocity resonance whereby the mechanical spring, plus any aerodynamic spring, balances out the model inertia. The only torque then required to oscillate the model at that particular frequency is equal to that due to aerodynamic damping. The strain-gage balance which is forward of all the bearings and other friction-producing devices only senses the aerodynamic damping. A system of resolvers, filters, and damped digital voltmeters is used to separate the torque signal into components in-phase and out-of-phase with the roll displacement. This makes it possible to measure the damping at different frequencies, but the most accurate method of operation is to oscillate at velocity resonance where the resultant moment measured by the strain gage is in-phase with the roll velocity.

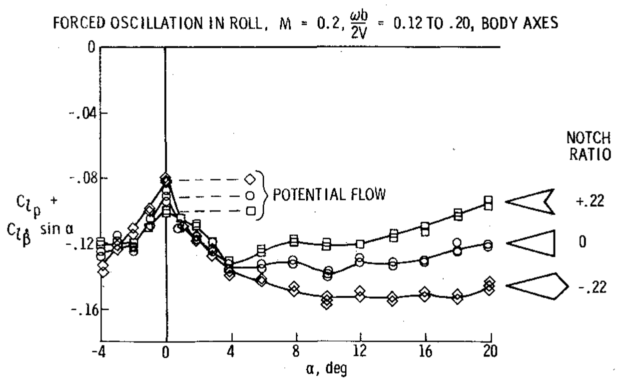

Fig. 9 Variation of damping-in-roll parameter with angle of attack for 74° sweep planforms.

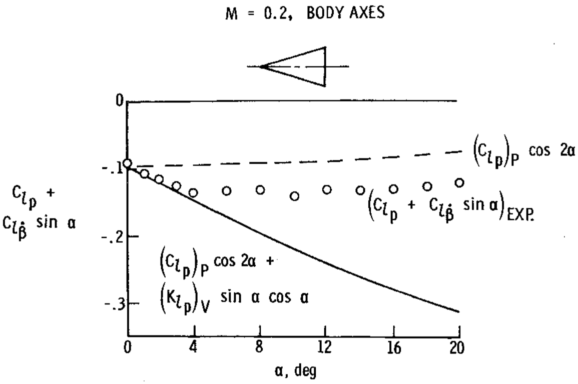

Fig. 10 Comparison of theory and experiment for 74° sweep delta wing.

Oscillatory Roll Tests of 74° Sweep Wings

Figure 9 illustrates some results recently obtained with the forced-oscillation roll mechanism for a series of 74° leading-

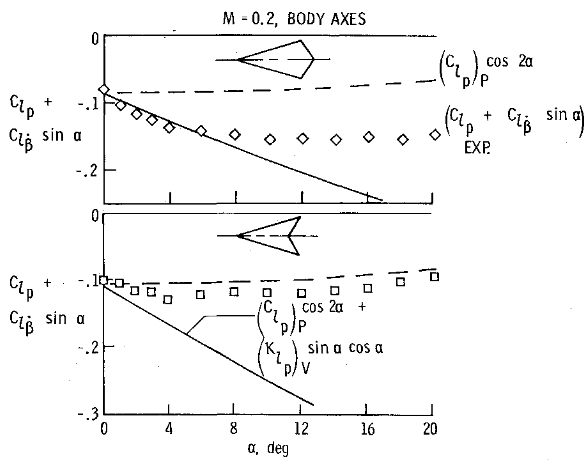

Fig. 11 Comparison of theory and experiment for 74° sweep diamond and arrow wings.

edge sweep wings with delta, arrow, and diamond planforms. All of the wings had a flat-plate airfoil section with sharp leading and trailing edges. A small cylindrical fairing was used to cover the balance and sting. The wings were designed specifically to investigate the effect of the leading-edge vortex flow on the roll damping of slender planforms of interest for supersonic vehicles. The results were determined at Mach numbers of 0.2 to 0.8 in the Langley 7 × 10-ft high-speed tunnel over an angle-of-attack range from −4° to +20°. Typical results are shown in Fig. 9 for a Mach number of 0.2 where the damping-in-roll parameter obtained about the model body axis is $C_{l_p} + C_{l_{\dot{\beta}}} \sin\alpha$. This combination of derivatives is a result of the roll displacement about the body axis combining with the angle of attack to produce a sideslip angle. This type of rolling motion is typical of slender configurations which have low roll inertia. The experimental magnitude of the damping-in-roll parameter is seen to increase with angle of attack in the low angle-of-attack range for all of the planforms which were tested. This increase in the damping-in-roll parameter is attributed to the leading-edge vortex lift which characterizes slender sharp-edge wings. The theoretical values of roll damping at 0° angle of attack shown on the figure were obtained from the modified Wagner lifting-surface theory. These predicted values, which are only for nonseparated flow, indicate approximately the same levels of roll damping that were experimentally measured at 0° angle of attack. At angles of attack greater than about 4°, the variation of the experimental damping-in-roll parameter with wing planform was different from that measured at low angles or calculated by attached flow theory. This is believed to be associated with differences in the vortex-flow induced effects over the aft portion of the wings. For example, the diamond planform has area aft of the wing tip which can be acted on by the leading-edge vortex to increase the vortex rolling moment while the arrow wing has much less area on which the vortex can act. The reduced damping of the arrow wing at the higher angles of attack is analogous to the pitchup problems of this wing planform.

Figure 10 compares the experimental damping-in-roll parameter ($C_{l_p} + C_{l_{\dot{\beta}}} \sin\alpha$) for the 74° delta wing with the roll damping (C_{l_p}) predicted by the vortex-flow theory in body axes. The $C_{l_{\dot{\beta}}}$ term is not included in the theoretical values as there is no adequate prediction method for $C_{l_{\dot{\beta}}}$ for wings. Even so, the agreement is good up to about 4° angle of attack. Beyond 4°, the experimental data fall between the potential-flow values and the vortex theory. The vortex-flow prediction forms an apparent upper limit on the roll damping. The lack of agreement at the higher angles of attack is thought to be caused by a combination of the inboard movement of the vortex cores with angle of attack, as previously discussed, and the $C_{l_{\dot{\beta}}}$ contribution which, because of the "sinα" multiplier, would become more important at the higher angles of attack.

The comparisons of the theory and the experimental data for both the diamond and arrow planforms are illustrated in Fig. 11. The vortex-flow theory underpredicts for the diamond planform and overpredicts for the arrow planform in the low angle-of-attack range. The effect of the additional area of the diamond planform as compared to the arrow planform is readily apparent in the angle-of-attack range from 4° to 20°. In this range, the arrow planform experimental data closely follow the potential theory, but this is considered coincidental since it is known that the flow is separated at the leading edge. The diamond planform shows good agreement with the vortex theory up to about 7° angle of attack and then falls off in the same manner as the delta planform data.

Conclusions

The leading-edge suction analogy, which has proven useful for lift and drag predictions, has been extended in an effort to understand the vortex-flow effects on the roll damping of slender wings at subsonic speeds. Comparisons with experiment indicate that the method can be useful in the prediction of the large increase in roll damping with angle of attack associated with the leading-edge vortex flow of highly swept, low-aspect-ratio wings in the design stage before actual dynamic-stability tests are made.

In addition, the newly developed forced-oscillation roll mechanism, suitable for dynamic-stability testing at subsonic and transonic Mach numbers, has been used to obtain roll-damping data on a series of 74° leading-edge sweep wing planforms. These data illustrate the effects of vortex flow on the damping-in-roll parameter of delta, arrow, and diamond planforms.

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